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Engineering Notes

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Numerical Simulation of Wrinkles in Space Inflatable Membrane Structures

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Nomenclature

B	=	strain-displacement matrix
C	=	damping matrix
f_{int}	=	vector of the internal forces
M	=	mass matrix
m_{ii}	=	diagonal term of the mass matrix related to the DOF i
$p(t)$	=	vector of the external forces
$\dot{u}$, $\ddot{u}$	=	vectors of the generalized velocities and accelerations
α	=	damping ratio
Δt	=	time step length
ξ	=	fraction of critical damping in the mode with the highest natural frequency
σ	=	vector of Cauchy stresses
$\omega_{\max}$	=	highest natural frequency of the finite element model

I. Introduction

ULTRALIGHTWEIGHT space inflatable structures have become very attractive because they can meet the structural requirements for space application at a low cost. These inflatable structures are often partially wrinkled and the formation of wrinkles drastically degrades the structural performance and the surface precision. A better understanding of the effects of the wrinkles on the structural performance of these space structures is essential and desirable. We developed a space inflatable reflector, which is shown in Fig. 1, and the study on wrinkles in this paper is based on this structure.

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The numerical simulation of the wrinkles in the membrane structures has attracted much interest in the past, starting from the development of the iterative membrane properties (IMP) method based on the tension field theory [1,2]. The IMP method, based on the membrane element, allowed the extent of the wrinkled region, the wrinkle angle, and the stress state within the membrane. One drawback on the use of this method is that the stability of the algorithm is poor. And the detailed out-of-plane wrinkle cannot be obtained by using this method. The detailed information about individual wrinkles, including the wrinkle amplitudes and wavelength, can be predicted using the nonlinear buckling finite element method, based on the thin shell element [3–8]. The convergence of this method is improved by introducing the artificial damping. However, it requires a highly mesh density to ensure the analysis accuracy. In addition, both methods are based on the static analysis technique, and they cannot be applied to the analysis of the dynamic deformation of wrinkles. The professional simulation of the dynamic deformation of the wrinkles in the space inflatable structure is correspondingly more complex, and relative reports are also absent [9–11].

In this paper, the explicit time integrate method incorporating the model of AIRBAG in LS-DYNA is conducted to the analysis of the dynamic deformation of wrinkles with time, including the occurrence and the evolution. The self-contact and damping in the AIRBAG model is used to ensure the convergence, and the effect of mesh density on the characteristic of wrinkles is also investigated.

II. Explicit Time Integration

The equations of motion of a discretized nonlinear structural system can be written as

$$M\ddot{u} + C\dot{u} + f_{\text{int}}(u) = p(t) \tag{1}$$

The internal forces include the effects of material and geometric nonlinearities. So, the internal force vector has to be updated at each time step during the time integration of the equation of motion. At the current configuration, the internal force may be evaluated from the expression $f_{\text{int}} = \int_V B^T \sigma \, \mathrm{d}V$, where V is the current volume.

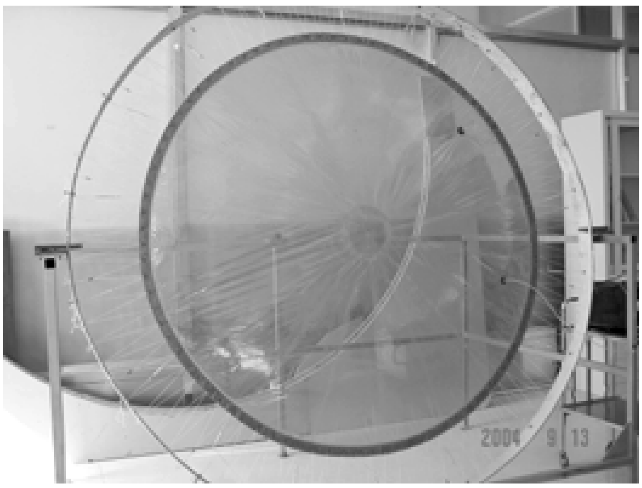


Fig. 1 Space inflatable reflector model.

A. Central Difference Method

The central difference method to approximate $\dot{u}$ and $\ddot{u}$ for integrating the equation of motion is given by

$$\dot{u}^t = \frac{1}{\Delta t}\left(u^{t+\frac{\Delta t}{2}} - u^{t-\frac{\Delta t}{2}}\right) \qquad \ddot{u}^t = \frac{1}{\Delta t}\left(\dot{u}^{t+\frac{\Delta t}{2}} - \dot{u}^{t-\frac{\Delta t}{2}}\right) \tag{2}$$

Substituting Eq. (2) into Eq. (1), and employing the lumped mass M and the damping $C = \alpha M$ matrices, the explicit solution of Eq. (1) can be given by the next expression,

$$\begin{aligned} \dot{u}_i^{t+\frac{\Delta t}{2}} &= \frac{2-\alpha\Delta t}{2+\alpha\Delta t} + \dot{u}_i^{t-\Delta t} + \frac{2\Delta t}{m_{ii}(2+\alpha\Delta t)}(p_i^t - f_{\mathrm{int}i}^t) \\ u_i^{t+\Delta t} &= u_i^t + \Delta t \dot{u}_i^{t+\frac{\Delta t}{2}} \end{aligned} \tag{3}$$

where $f_{\mathrm{int}i}^t$ is the internal force related to the degree of freedom (DOF) i, and p_i^t is the given external force related to the same DOF i.

B. Stability of Central Difference Method

The central difference method is stable for a time step increment satisfying the relationship $\Delta t \leq 2/\omega_{\max}$. An upper bound for the stable time step with damping as the critical time step can be expressed as

$$\Delta t \leq \frac{2}{\omega_{\max}}\left(\sqrt{1+\xi^2} - \xi\right) \tag{4}$$

Thus, damping reduces the critical time step size. The time step size is bounded by the largest natural frequency of the structure, which in turn is bounded by the highest frequency of any individual element in finite element mesh.

III. Numerical Simulation of Wrinkles in Inflatable Structures

The dynamic wrinkle deformation and the stress state in the hyperbolic paraboloidal inflatable structure are analyzed by using the explicit method incorporating the nonlinear dynamic solver of the LS-DYNA code. The material performances and the finite element model are shown in Fig. 2. The model is free and the mass flow rate is 10 kg/s. The paraboloidal equation is $y + 117.1875 = (x^2 + z^2)/4800$.

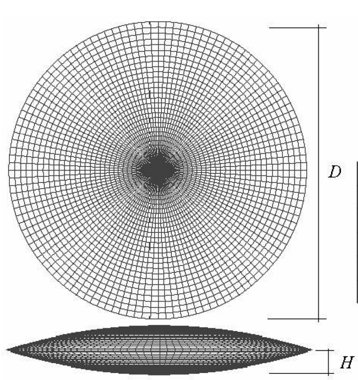
Figure 3 is the result of the wrinkled deformation for time 0.018 s. The large wrinkles occur at the outer edge of the model, and the wrinkle amplitude is similar to each other. The relationship between the time and the nodal horizontal displacement of the large wrinkle, node 2652, is plotted in Fig. 4.

Observed in Fig. 4, the wrinkled amplitude is increasing and then decreasing with the increment of time, which indicates the evolution of the wrinkles in the model. The maximum displacement of node 2652 is 25.34 mm, which is considered as the large wrinkle amplitude.

The contours of von Mises stress, the maximum and minimum principal stress of element 24501 in the large wrinkle are shown in Figs. 5a–5c. The element 24501 contains node 2652. According to Fig. 5, the maximum principal stress for $t = 0.018$ s is 12.64 MPa, and the minimum principal stress for $t = 0.018$ s is -0.286 MPa.

Mesh density is one of the most important factors, which affects the precision of the analysis. In this section, three different mesh sizes were used to investigate the effect of mesh density on the characteristic of wrinkles in inflatable structures. The effects of mesh refinement on the nodal displacement, maximum principal stress, and minimum principal stress are shown in Figs. 6a–6c, respectively.

It was observed in Fig. 6 that there is no obvious change in the displacement and principal stress plots, when starting from the reference mesh with 20,000 elements in the model; the number of elements is roughly doubled. Therefore, we can conclude that the reference mesh is sufficiently fine to produce mesh-independent results. Because the computational time increases roughly proportional to the number of elements, it would be pointless to use a mesh finer than the reference one. The sudden jumping of the stress curve revealed the change and development of the wrinkles. According to Fig. 6, the wrinkles in the structure are fluctuant and unstable in the evolution.



Aperture, D	1500mm
Depth of paraboloid, H	117.1875mm
Thickness, h	0.025mm
Young's modulus, E	3.53GPa
Density, ρ	1420Kg/m³
Poisson's ratio, ν	0.3

Fig. 2 Finite element model of hyperbolic paraboloidal inflatable structure.

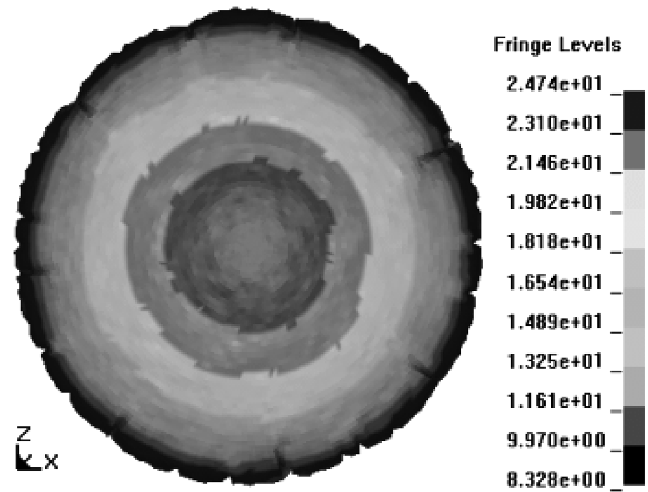


Fig. 3 Wrinkled deformation on an inflatable paraboloidal airbag.

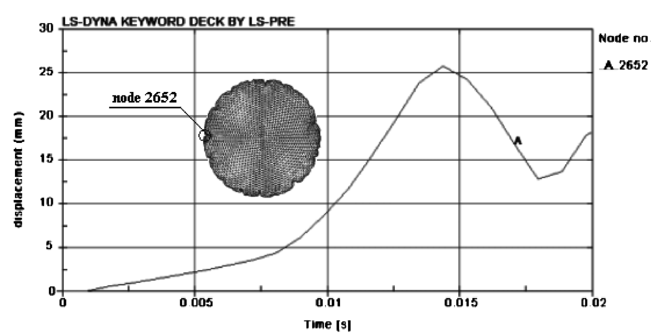


Fig. 4 Time-displacement curve of node 2652 in the large wrinkle.

IV. Experimental Verification

To verify the ability of the numerical model to predict the wrinkles in an inflatable airbag, a simple wrinkle experiment is carried out. The experimental instrument is an air compressor with adjustable inflation pressure and velocity. The material used in experiments is Kapton, and material performances can be observed in Fig. 2. The paragraph of the inflation experiment is shown in Fig. 7.

According to the experiment, we can observe that the pattern and the region of wrinkles in experiments show good agreement with the results of simulation. The experimental average wrinkle amplitude is 20.1 mm, and it is slightly less than the numerical result.

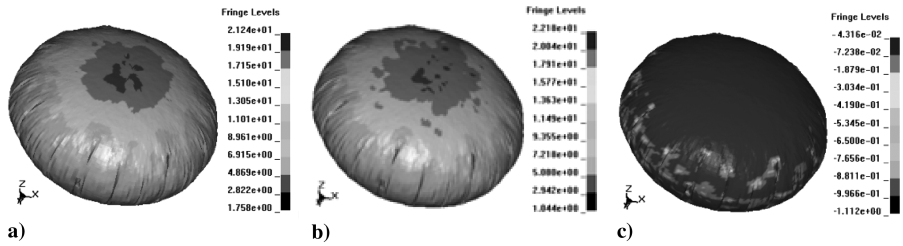


Fig. 5 The stress distribution in a wrinkled membrane structure. a) von Mises stress, b) maximum principal stress, and c) minimum principal stress.

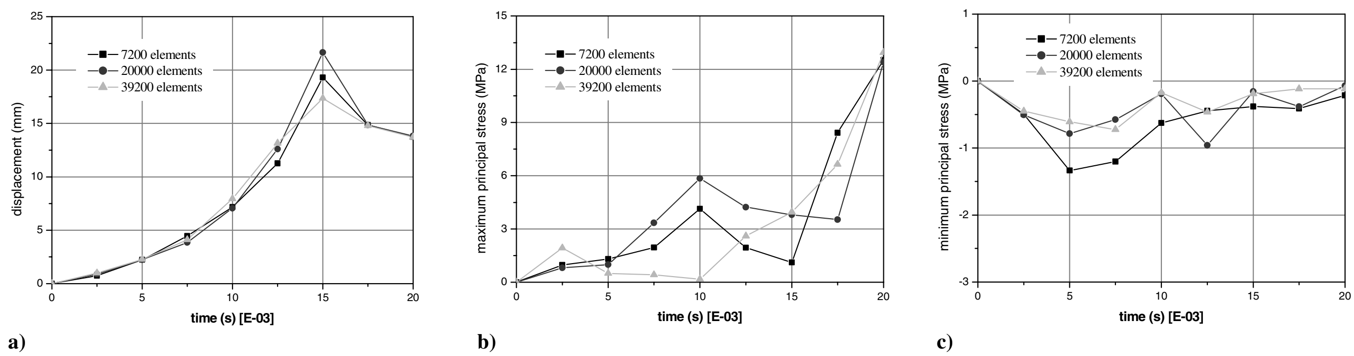


Fig. 6 Effects of mesh refinement on the analytical results. a) wrinkle displacement, b) maximum principal stress, and c) minimum principal stress.

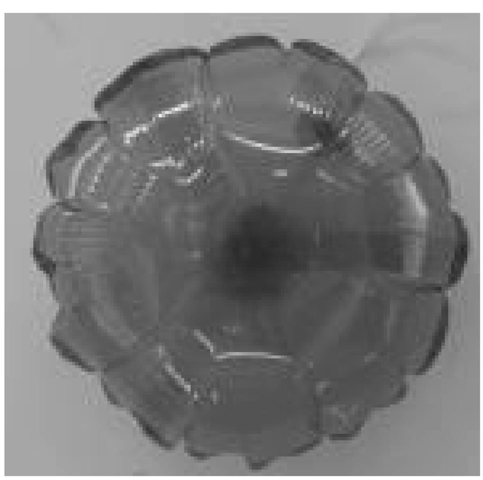


Fig. 7 Paragraph of inflation experiments.

V. Conclusion

The finite element simulation technique, based on the explicit time integration method, presented in this paper has been shown to be robust and capable of producing good quality results, and numerical results can accurately simulate the physical experiment results. There is no obvious change in the analytical results for the different mesh density, and the reference mesh, 20,000 elements in the model, is sufficiently fine to produce accurate results. The sudden jumping of the stress curve revealed the fluctuate evolution of the wrinkles, and the discontinuous points represented the occurrence of the new wrinkles. The analytical results can be used to support the structural design of inflatable structures.

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